Diffraction of Light

Zone Plate

A zone plate is a special diffracting screen designed to obstract light from alternate half period zones.

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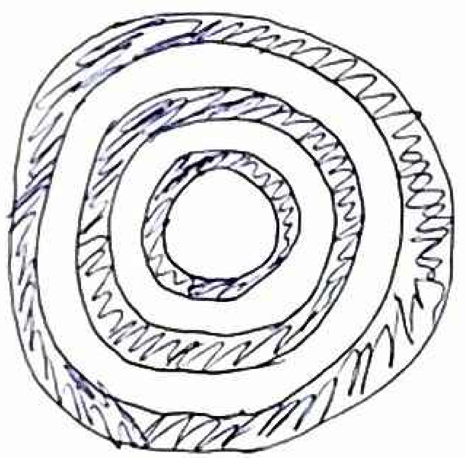
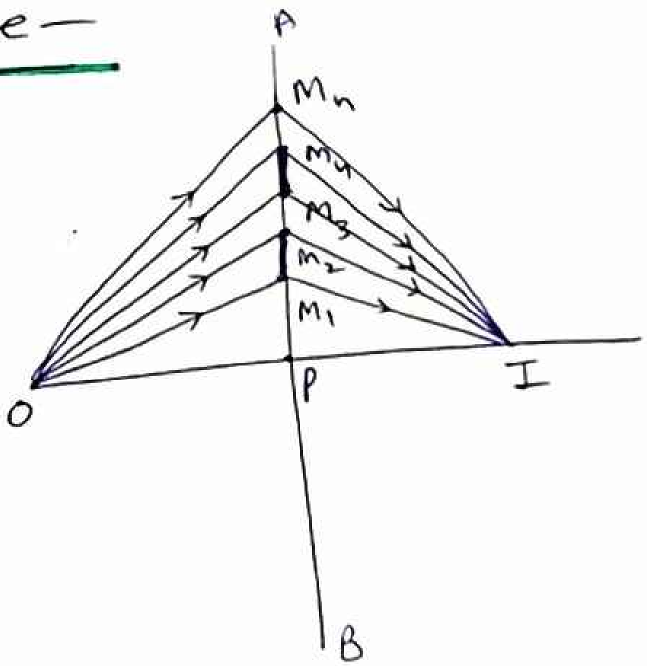
Diffraction of light

It is constructed by drawing a series of concentric circles on a sheet of white paper with radii $\propto \sqrt{\text{natural numbers}}$. Alternate zones are painted black.

A highly reduced photograph of this drawing is then taken. The negative of this photograph is called "zone plate". It behaves like a convex lens.

Theory of zone plate—





Suppose AB is a transparent plate, let a point source be placed at point O such that $\perp$ distance $OP = u$. We have to find the intensity of light at point I, where $PI = v$.

Now, let us draw concentric ~~coil~~ circles on the plate, taking P as the centre and radii equal to $PM_1 = r_1$, $PM_2 = r_2$, $PM_n = r_n$ etc., such that

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$$OM_1I - OPI = \lambda/2$$

$$OM_2I - OPI = 2\frac{\lambda}{2}$$

$$\boxed{OM_nI - OPI = \frac{n\lambda}{2}} \quad \text{(1)}$$

Thus the path diff. between the waves from consequtive zones will be $\frac{\lambda}{2}$ when they reach the point I. Thus the plate will be divided in to First half period zone, Second half period zone and so on.

Now, let us calculate the radius r_n of the n^{th} zone. From ΔOPM_n,

$$(OM_n)^2 = (PM_n)^2 + (OP)^2$$

$$\text{or } OM_n = (r_n^2 + u^2)^{1/2}$$

$$= u\left(1 + \frac{r_n^2}{u^2}\right)^{1/2}$$

$$= u\left[1 + \frac{r_n^2}{2u^2}\right], \text{ to the first approximation}$$

$$= u + \frac{r_n^2}{2u}$$

Similarly, $IM_n = v + \frac{r_n^2}{2v}$

$$\therefore OM_nI - OPI = OM_n + M_nI - OPI$$

$$= u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} - (u+v)$$

$$= \frac{r_n^2}{2u} + \frac{r_n^2}{2v} = \frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right)$$

$\therefore$ from eq. (1), $\frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{n\lambda}{2}$ — (2)

$$\text{or} \quad r_n^2 = \frac{uv}{(u+v)} n\lambda$$

$$\Rightarrow \boxed{r_n \propto \sqrt{n}}$$

Thus if we want plate AB to be divided in to half period zones, radii of the circles must be proportional to the square root of the natural numbers.

Now, if the alternate zone are made opaque, the plate will ~~sere~~ serve as "zone plate". Also, as we have seen that the amplitude at I due to a zone decreases as the order of zone increases.

Further, as the waves from the successive transparent zones differ in path-length by λ, they reach I ~~at~~ in the same phase.

∴ Total amplitude at I,

$$R = R_1 + R_3 + R_5 + \cdots$$

which is greater than $R_1/2$ (which is the amplitude at I, if all the zones are transparent). Hence point I has sufficient light and we may call it as the image of O. Thus zone plate focuses light from O at I. Thus it behaves like a convex lens.

Now from eq. (2), $\frac{1}{v} + \frac{1}{u} = \frac{n\lambda}{r_n^2}$

Comparing it ~~which~~ with the lens formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, we get $f = \frac{r_n^2}{n\lambda}$

which gives the focal length of the zone plate.

Multiple foci of Zone plate — Now suppose we want to find the intensity of light at point I_3 such that $v_3 = \frac{r_n^2}{3n\lambda}$ (point O remaining at u from AB).

From eq. (2), If $u = \infty$ then

$$v = \frac{r_n^2}{n\lambda} = f$$

$$\Rightarrow v_3 = \frac{v}{3}$$

$\Rightarrow$ each zone will now contain three new zones corresponding to point I_3. ($\because$ area of a zone $= p\pi\lambda = v\pi\lambda$, and now v has become one third)

$\therefore$ Resultant amplitude at I_3 will be

$$S = (s_1 - s_2 + s_3) - (s_4 - s_5 + s_6) + (s_7 - s_8 + s_9) - (s_{10} - s_{11} + s_{12}) + \cdots$$

$$= (s_1 - s_2 + s_3) + (s_7 - s_8 + s_9) + (s_{13} - s_{14} + s_{15}) + \cdots$$

$$= \left[\left(\frac{s_1}{2} - s_2 + \frac{s_3}{2}\right) + \left(\frac{s_1}{2} + \frac{s_3}{2}\right)\right] + \left[\left(\frac{s_7}{2} - s_8 + \frac{s_9}{2}\right) + \left(\frac{s_7}{2} + \frac{s_9}{2}\right)\right] + \cdots$$

$$= \frac{s_1 + s_3}{2} + \frac{(s_7 + s_9)}{2} + \frac{(s_{13} + s_{15})}{2} + \cdots$$

or $s = \frac{1}{2}(s_1 + s_3 + s_7 + s_9 + \cdots)$

~~spot~~ Thus point I_3 receives sufficient light though less than I. Hence I_3 is another image of O. Similarly it can be shown that point I_5, I_7, I_9 etc such that

$$v_5 = \frac{r_n^2}{5n\lambda}, \quad v_7 = \frac{r_n^2}{7n\lambda} \text{ etc.}$$

are the images of O. Hence zone plate has multiple foci with focal lengths –

$$\frac{r_n^2}{n\lambda}, \quad \frac{r_n^2}{3n\lambda}, \quad \frac{r_n^2}{5n\lambda} \text{ etc.}$$